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Experiments on coherence resonance: Noisy precursors to Hopf bifurcations

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Experimental and numerical evidence of coherence resonance in an electrochemical system is reported. External noise with a Gaussian distribution is superimposed on the system when the anodic current is exhibiting stationary (fixed point) dynamics below a supercritical Hopf bifurcation. The amplitude of the added stochastic perturbations is increased monotonically and the induced oscillatory behavior is analyzed. It is observed, both in experiments and in simulations, that the regularity of the noise induced current oscillations reaches a maximum value for an optimum noise level. This is indicative of coherence resonance and can be explained with a mechanism based on noisy precursors to a Hopf bifurcation.

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The apparent counterintuitive ability of noise to increase the coherence of a dynamical system has attracted the attention of the scientific community. This phenomenon can be roughly classified into two different areas. First, stochastic resonance (SR) [1] is the term used to define a process in which the addition of fluctuations amplifies a weak deterministic signal. An optimum amplification (intensity and fidelity) of this signal is achieved for a particular level of superimposed noise amplitude. SR has been detected and analyzed [2] in a number of physical [3], chemical [4,5], and biological [6] systems. Second, coherence resonance (CR) [7–9] is the emergence of coherence in noise-induced oscillations. CR differs from SR in that in the former there is no deterministic signal being enhanced by virtue of superimposed noise. CR can manifest itself in different systems due to different mechanisms. In excitable systems, the essential role of slow and fast dynamics, and the nonuniformity of oscillations was emphasized [8,10]. This approach was successful in interpreting CR in neuron models [11,12] and in the Yamada model of a semiconducting laser [13]. CR in excitable systems was experimentally confirmed in a laser diode with optical feedback [14] and in electronic circuits [15]. CR has also been observed in bistable systems such as in simulations of the FitzHugh-Nagumo model [12] and in both simulations and experiments with the chaotic Chua model [16].

A general mechanism of CR was proposed based on "noisy precursors" [17]. Wiesenfield demonstrated [18] that the power spectrum of a system observed after a bifurcation point can, nevertheless, be visible even before the bifurcation point if there is noise present. Coherence resonance can be explained [17] by interplay of the constructive effect of noise as promoting coherent precursors and by the well-known destructive effect as eliminating order.

In this article, we provide experimental confirmation of coherence resonance in a system that is neither excitable nor bistable. We consider the effect of noise on a stable focus in the vicinity of a supercritical Hopf bifurcation; we investigate the noisy precursors to the bifurcation. The periodicity of the noise provoked oscillations is quantified using the coherence factor β [7,19],

$$\beta = H/W, \tag{1}$$

where H is the height and W is relative width of the dominant peak of the power spectrum. The experimental work is augmented by supporting numerical simulations.

A standard electrochemical cell consisting of a nickel working electrode (Aldrich, 99.99%+, 1 mm diameter), a Hg/Hg_2SO_4/cc. K_2SO_4 reference electrode, and a platinum mesh counter electrode are used in the experiments. The electrode is embedded in epoxy and reaction takes place only at the ends. The potential of the electrode V is controlled with a potentiostat (EG&G PAR 273). Noisy perturbations of the circuit potential $V(t) = V_0 + D\xi(t)$, where V_0 is an offset potential (set point of the autonomous system), D is noise amplitude, and $\xi(t)$ is zero mean Gaussian noise with unit standard deviation are carried out with 16-bit D/A card with an update frequency of 100 Hz and accuracy 0.03 mV. Zero resistance ammeter is used to measure the current of the electrode (rate of Ni dissolution) and data acquisition is done at 100 Hz. The experiments are carried out in 4.5 M sulfuric acid solution at a temperature of 11 °C.

The dynamics of the electrodissolution of Ni in sulfuric acid have been known to exhibit rich dynamics including bistability, oscillations generated by Hopf and saddle-loop bifurcations, and low-dimensional chaos [20,21]. The various behaviors are obtained with different experimental conditions such as concentration of electrolyte, circuit potential, mode of cell operation (galvanostatic or potentiostatic), and external resistor connected in series with the electrode. To obtain coherence resonance, we set the conditions (4.5 M H_2SO_4, 250 Ω resistor) close to a supercritical Hopf bifurcation. The primary bifurcation parameter is the circuit potential. At $V_0 = 1.155$ V, a supercritical Hopf bifurcation takes place and oscillations with a frequency of about 2.2 Hz are seen above the bifurcation point. The experiments are carried out in the suboscillatory region $V_0 = 1.153$ V and

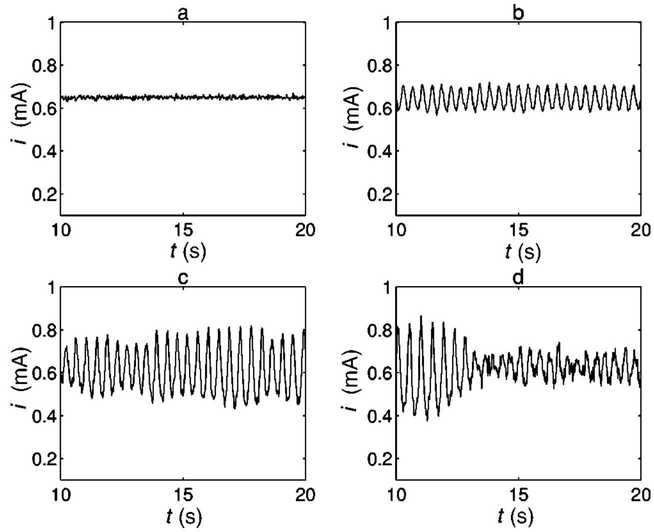

FIG. 1. Experiments. Time series of current at different noise strengths ($V_0 = 1.153$ V, $R = 250\ \Omega$). (a) Without noise, $D = 0$ mV. (b) Small added noise, $D = 3$ mV. (c) Optimal coherence at intermediate noise strength, $D = 12$ mV. (d) Large noise amplitude, $D = 18$ mV.

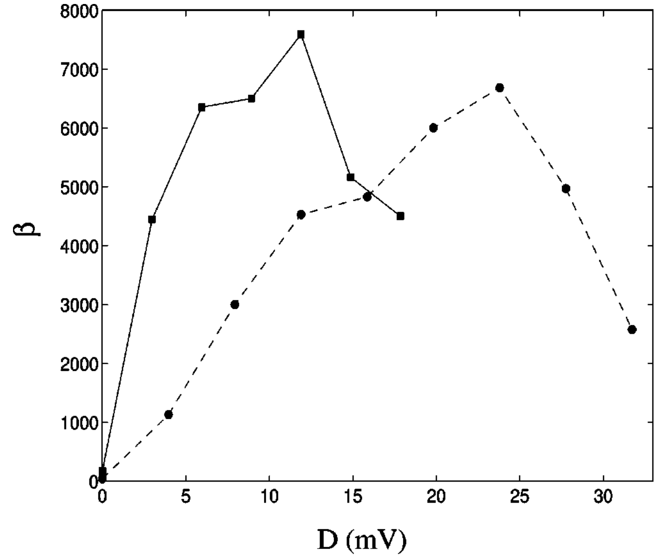

FIG. 2. Experiments. The coherence factor β as a function of noise amplitude D for just below ($V_0 = 1.153$ V, squares) and further away ($V_0 = 1.148$ V, circles) from the Hopf bifurcation point.

$V_0 = 1.148$ V, i.e., 2 and 7 mV below the supercritical Hopf bifurcation point.

A series of experiments have been performed (in the suboscillatory parameter domain) by increasing the noise strength D on circuit potential. Results shown in Fig. 1 correspond to $V_0 = 1.153$ V, i.e., just below the Hopf bifurcation. Without any noise, the system rests at steady state [see Fig. 1(a)]. As a result of some very small inherent experimental noise, the steady state of the system is continuously disturbed and a somewhat noisy fixed point behavior is observed. The power spectrum analysis shows a small peak at $f = 2.58$ Hz, which can be regarded as an indication of noisy precursors due to weak inherent system noise. This frequency is close to that of the autonomous oscillations observed after the Hopf bifurcation (2.2 Hz). With small added (external) noise, small amplitude oscillations start to emerge [Fig. 1(b)] with a frequency $f = 2.55$ Hz. With increasing noise, the amplitude of the oscillations increases [Fig. 1(c)]. With excessive noise, intermittent regions of noisy periodic oscillations and noise dominated regions are observed as is shown in Fig. 1(d).

The coherence factor [7,19] was calculated to quantify the periodic nature of the noise induced oscillations in the anodic current. Figure 2 shows the computed coherence factor curves using the experimental time series. The two curves correspond to two different values of V_0 in the suboscillatory regime. The coherence factor exhibits a maximum as a function of noise amplitude. The position of the maximum depends on the distance of the set point V_0 from the Hopf bifurcation. Optimal coherence occurs at smaller noise amplitude close to the Hopf bifurcation.

We used the following dimensionless model [20] simulating Ni dissolution to validate our experimental findings:

$$\frac{de}{dt} = \frac{V - e}{R} - \left[C_h \frac{\exp(0.5e)}{1 + C_h \exp(e)} + 0.3 \exp(e) \right] (1 - \theta), \tag{2}$$

$$\Gamma_1 \frac{d\theta}{dt} = \frac{\exp(0.5e)}{1 + C_h \exp(e)} (1 - \theta) - 6 \times 10^{-5} C_h \frac{\exp(2e)}{10^{-3} C_h + \exp(e)} \theta, \tag{3}$$

where e is the dimensionless potential drop on the double layer, θ is the total coverage of nickel oxide and hydroxide, C_h is the dimensionless concentration of the electrolyte, Γ_1 is a time scale ratio, V is the dimensionless circuit potential, and R is the dimensionless series resistance. The dimensionless current is obtained as $i = (V - e)/R$. This two-dimensional model captures the essential features of the experimental electrochemical cell used in experiments.

Dynamics similar to those seen in experiments are obtained with model parameters $\Gamma_1 = 10^{-2}$, $C_h = 1600$, and $R = 5$. As the bifurcation parameter V is continuously increased, the model system undergoes a supercritical Hopf bifurcation at $V_{\mathrm{Hopf}} = 2.867$. Oscillations in the dimensionless anodic current with a natural frequency of $f = 8.94 \times 10^{-2}$ occur for $V_0 = 3.0$, above the Hopf bifurcation. Similar to the experiments, simulations are carried out in the suboscillatory region, $V_0 = 2.7$ and $V_0 = 2.0$, below the supercritical Hopf bifurcation point.

To mimic experimental results the circuit potential was perturbed stochastically $V = V_0 + D\xi$. This system of stochastic differential equations was integrated using the Euler-Maruyama algorithm [22]. The Euler-Maruyama method is a simple stochastic Euler integrator used to simulate noisy differential equations. Figure 3(a) shows the time series ($V_0 = 2.7$) without noise. The model system exhibits stable focus

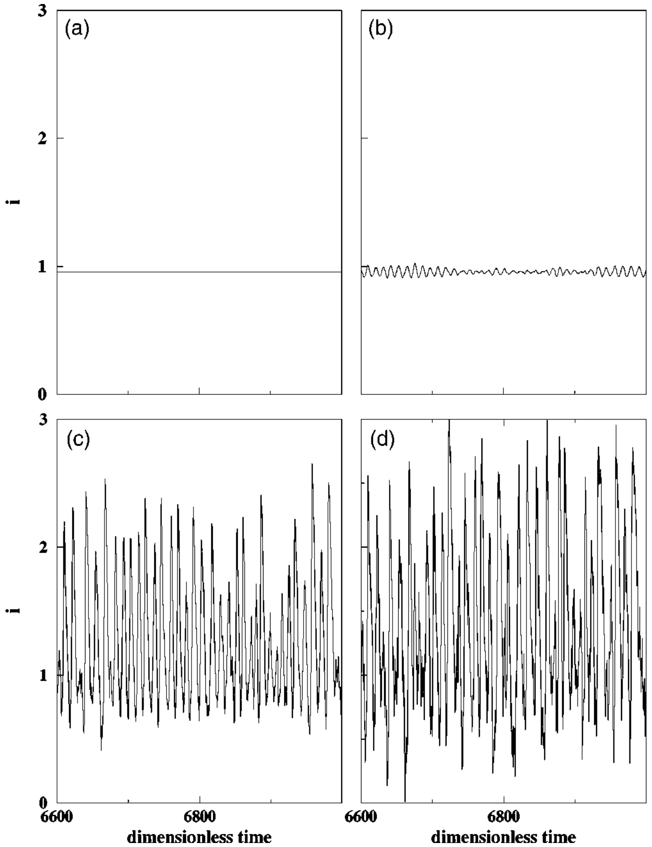

FIG. 3. Simulations. Time series of dimensionless current at different noise strengths ($V_0=2.7, R=5$). (a) Without noise (stable focus), $D=0$. (b) Small noise, $D=0.02$. (c) Optimal coherence at intermediate noise strength, $D=0.55$. (d) Large noise amplitude, $D=1.0$.

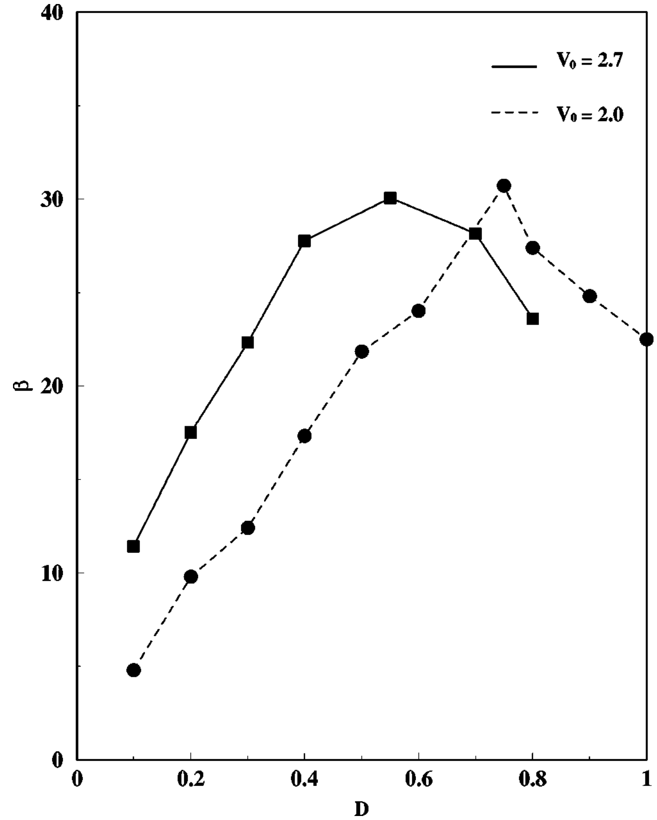

FIG. 4. Simulations. The coherence factor β as a function of noise amplitude D for just below ($V_0=2.7$, squares) and further away ($V_0=2.0$, circles) from the Hopf bifurcation point.

dynamics with a characteristic frequency of $f=9.27\times10^{-2}$ in the vicinity of the stable fixed point. With small added noise small amplitude oscillations start to emerge [Fig. 3(b)] with a frequency $f=9.22\times10^{-2}$ close to the characteristic frequency of the stable focus, and also close to the frequency of oscillations above the Hopf bifurcation. Figures 3(c) and 3(d) show the noise provoked oscillations for optimum and large amounts of noise. These invoked current oscillations are consistent with the experimental observations shown in Fig. 1.

Figure 4 shows the coherence factor β computed for two different set points V_0 (in the suboscillatory domain) of the unperturbed dynamics. Consistent with experimental observations, the curves show maxima as the amplitude of superimposed noise is increased. This indicates that for an optimum value of added noise maximal regularity of the induced current oscillations is achieved. As in experiments, our simulations indicate that for set point V_0 closer to the supercritical Hopf bifurcation maximum β occurs at smaller noise amplitude.

We have carried out laboratory experiments on coherence resonance close to a supercritical Hopf bifurcation. The emergence of noise induced oscillations has been observed; the regularity of these induced oscillations is quantified using the coherence factor β. It is shown that the induced current oscillations show maximal periodicity for a particular amplitude of the superimposed noise. The coherence resonance can be understood based on the idea of noisy precursors; the frequencies of the induced oscillations are close to that of the autonomous oscillations above a neighboring bifurcation point. The experimental results were confirmed by numerical simulations of the chemical system.

The results on coherence resonance near a Hopf bifurcation can be extended to a variety of nonlinear systems. Evidence of coherence resonance indicates the emergence of structure (regularity) in the presence of random fluctuations. The self-organization manifested by numerous biological systems immersed in noisy environments could perhaps have a similar underlying mechanism.

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